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A Perturbation Solution for Fins with Conduction, Convection, and Radiation Interaction

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Nomenclature

A = cross-sectional area of fin
E = radiation emittance (total hemispherical value)
h = convective heat-transfer coefficient
k = thermal conductivity
L = length of fin
p = perimeter
T = temperature
X = axial distance, measured from fin base
Z = dimensionless distance = X/L
ϵ = radiation-conduction parameter = $\sigma EpL^2T_b^3/kA$
θ = dimensionless temperature = T/T_b
λ^2 = Biot modulus = hpL^2/kA
ν = fin geometry parameter = A/pL
σ = Stefan-Boltzmann constant

Subscripts

b = fin base
e = environment

Introduction

INTEREST in space radiators has stimulated considerable work on conduction problems with surface radiation. Such problems are characterized by nonlinear differential equations as in fins, or nonlinear boundary conditions as in one-dimensional slabs. Because of the nonlinear nature of these systems most exact solutions involve numerical integrations requiring the use of computers.[1–4]

Perturbation techniques have been extensively applied in obtaining approximate solutions to problems in fluid mechanics. However, their utilization in solving heat-transfer problems involving the interaction of conduction with radiation has been limited. Tien and Abu-Romia[5] discuss the analysis of radiation interaction problems using perturbation methods. Dicker and Asnani[6] developed a perturbation technique for determining the transient temperature in a one-dimensional slab that is insulated on one side and subject to radiation on the other side. Mueller and Malmuth[7] applied the techniques of singular perturbation to solve the steady-state conduction problem with surface radiation and aerodynamic heating for a finite fin insulated on both ends.

This Note presents a regular perturbation solution to the fin problem with a constant base temperature and a radiating tip condition. The solution takes into consideration the interaction of conduction with radiation and convection. To examine the accuracy of the perturbation solution, a numerical solution is obtained.

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Analysis

The system under consideration is a constant cross-sectional area fin of length L and perimeter p, which is maintained at constant base temperature T_b at $X = 0$ and subject to radiation to an environment at temperature T_e at $X = L$. The fin also exchanges heat with the environment along its length by convection and radiation. The thermal conductivity k, radiation emittance E, and convective heat-transfer coefficient h are assumed constant. Based on these assumptions, the one-dimensional steady-state conduction equation, written in dimensionless form, is

$$d^2\theta/dZ^2 - \epsilon\theta^4 - \lambda^2\theta = -\epsilon\theta_e^4 - \lambda^2\theta_e \tag{1}$$

with boundary conditions

$$\theta(0) = 1 \tag{2}$$

$$d\theta(1)/dZ = -\epsilon\nu[\theta^4(1) - \theta_e^4] \tag{3}$$

where

$$\epsilon = \sigma EpL^2T_b^3/kA \text{ (radiation-conduction parameter)} \tag{4}$$

$$\lambda^2 = hpL^2/kA \text{ (Biot modulus)} \tag{5}$$

$$\nu = A/pL \text{ (geometry parameter)} \tag{6}$$

The case of $\epsilon \ll 1$, which corresponds to weak radiation-conduction interaction, is considered. An asymptotic expansion for θ in the perturbation parameter ϵ is assumed. Considering the first-order approximation in ϵ, gives:

$$\theta(Z;\ \epsilon) = \theta_0(Z) + \epsilon\theta_1(Z) \tag{7}$$

Substituting (7) in Eqs. (1–3) and equating terms of equal powers of ϵ the zero- and first-order problems are obtained. For the zero order, the governing equation and boundary conditions are

$$d^2\theta_0/dZ^2 - \lambda^2\theta_0 = -\lambda^2\theta_e \tag{8a}$$

$$\theta_0(0) = 1 \tag{8b}$$

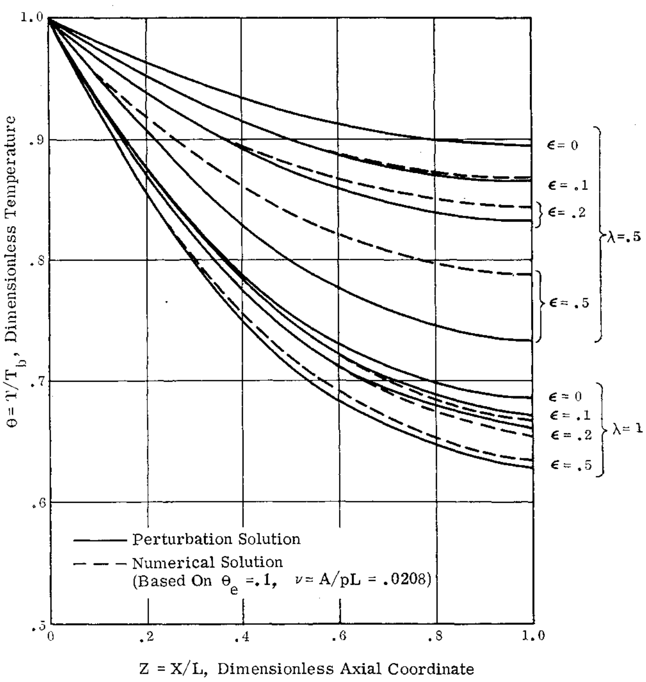

Fig. 1 Conductive fin subject to a radiative and convective environment.

$$d\theta_0(1)/dZ = 0 \tag{8c}$$

The first-order problem is

$$d^2\theta_1/dZ^2 - \lambda^2\theta_1 = -\theta_e^4 + \theta_0^4 \tag{9a}$$

subject to

$$\theta_1(0) = 0 \tag{9b}$$

$$d\theta_1(1)/dZ = -\nu[\theta_0^4(1) - \theta_e^4] \tag{9c}$$

The zero-order problem (8) is a nonradiating fin whose solution is

$$\theta_0(Z) = [(1 - \theta_e)/\cosh\lambda]\cosh\lambda(1 - Z) + \theta_e \tag{10}$$

The solution to the first-order problem (9) is

$$\theta_1(Z) = c_1e^{\lambda Z} + c_2e^{-\lambda Z} + \sum_{j=2}^{4} a_j \cosh j\lambda(1 - Z) + a_1 Z \sinh\lambda(1 - Z) + c \tag{11}$$

where

$$a_1 = -(4\theta_e\alpha/\lambda)(3\alpha^2 + \theta_e^2),\ a_2 = (2\alpha^2/\lambda^2)(\tfrac{4}{3}\alpha^2 + 2\theta_e^2)$$

$$a_3 = \theta_e\alpha^3/\lambda^2,\ a_4 = \tfrac{2}{15}\alpha^4/\lambda^2,\ c = -(6\alpha^2/\lambda^2)(\alpha^2 + 2\theta_e^2)$$

$$c_1 = c_2e^{-2\lambda} + \psi_2,\ c_2 = (\psi_1 - \psi_2)/(1 + e^{-2\lambda})$$

$$\alpha = \frac{1}{2}\left(\frac{1 - \theta_e}{\cosh\lambda}\right)$$

$$\psi_1 = -c - \sum_{j=2}^{4} a_j \cosh j\lambda$$

$$\psi_2 = e^{-\lambda}[a_1 - (8\nu/\lambda)(2\alpha^4 + 4\theta_e\alpha^3 + 3\theta_e^2\alpha^2 + \theta_e^3\alpha)]$$

The complete solution, expressed in terms of Eqs. (7, 10, and 11), gives the temperature profile along the fin in terms of the radiation-conduction parameter ϵ, convection parameter (Biot modulus) λ, environment temperature θ_e, and fin geometry ν. The simple form of the solution permits temperature calculations without recourse to a computer and enables evaluation of the effects that problem parameters $(\epsilon,\lambda,\theta_e,\nu)$ have on the response of the fin.

To check the accuracy of the perturbation solution, the problem was also solved numerically. Figure 1 gives temperature profiles for various values of the perturbation parameter ϵ and Biot modulus λ^2, for $\theta_e = 0.1$ and $\nu = 0.0208$. As expected, the perturbation solution becomes increasingly more accurate as the perturbation parameter ϵ decreases, since for small ϵ, the nonlinear radiation effects are small relative to conduction. The accuracy of the solution is also observed to improve as the Biot modulus λ^2 is increased. For large λ, radiation effects become small relative to convection and the accuracy of the perturbation solution again improves.

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Piston-Retardation Insert in a Free-Piston Compressor

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Introduction

FREE-PISTON compressors are commonly used for generating high temperatures and pressures in gases.[1,2] For shock-tube applications, the shock-tube driver gas is heated in the free-piston compressor and released into the driven shock tube by the opening of a high-pressure diaphragm. Due to experienced variations in diaphragm rupture pressures and difficulties in estimating peak pressures for given gas-loading conditions, the used diaphragms are allowed to break at pressures typically 20% lower than corresponding closed-end peak pressures. In this case, the piston possesses residual kinetic energy which has to be absorbed, normally as deformation work in some part of the mechanical structure. With incorrect initial gas loading or with a diaphragm weaker than normal, the residual energy could be larger than anticipated and mechanical destruction of the high-pressure section and the piston could result. In experiments with a free-piston compressor in a so-called bypass piston tube,[3] a sonic orifice has been inserted near the high-pressure diaphragm and successfully used in preventing such a high-speed impact. The insert, referred to as a piston-retardation insert, also favorably increases the entropy of the gas during the compression. Properly operating, e.g., in the case of an early rupture of the diaphragm, the piston will bounce in front of the insert or smoothly dock to the insert, with the gas between

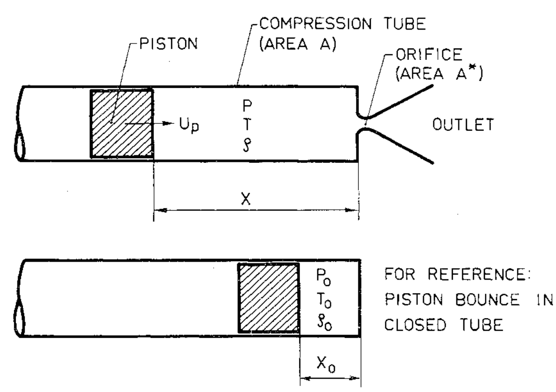

Fig. 1 Schematic view of the high-pressure section of a free-piston compressor with a sonic-orifice outlet. For reference is shown a piston bounce in a closed compression tube.

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